# Reducing Number of Elements of Transducer Arrays in Fourier Image Construction Method

Jian-yu Lu, Member, IEEE, and Shiping He, Member, IEEE
Ultrasound Laboratory, The University of Toledo,
Toledo, Ohio 43606, U.S.A. E-mail: jilu@eng.utoledo.edu

Abstract—In this paper[1], we study quantitatively the relationship between the quality of images constructed with the Fourier method and the element spacing of array transducers. In the study, two linear arrays were used. Effective larger element spacings were obtained by combining signals from adjacent elements. Both computer simulation and experiment were performed. Results show that resolution of constructed images is not affected by the reduction of number of elements, but the contrast of images is decreased dramatically when the element spacing is larger than about $2.365\lambda$, where $\lambda$ is the wavelength. This suggests that an array of about $2.365\lambda$ spacing can be used with the Fourier method. This reduces the total number of elements of a fully sampled $128\times128$ array ($0.5\lambda$ spacing) from 16384 to about 732.

## INTRODUCTION

Recently, a Fourier method has been developed to construct images [1-3] with limited diffraction beams [4-9]. This method can achieve a very high frame rate (about 3750 frames/s at a depth of about 200 mm in biological soft tissues) for either two-dimensional (2D) or three-dimensional (3D) imaging because only one transmission is required. The image signal-to-noise ratio (SNR) is also high because all array elements are used in transmission and the transmit beams do not diverge. In addition, imaging hardware for the new method can be greatly simplified. To implement the Fourier method, theoretically, an array transducer with an element spacing (distance between the centers of two adjacent elements) of about $\lambda/2$, where $\lambda$ is the wavelength, is required [1,2]. However, such an array has a large number of elements, especially, for a 2D array in 3D imaging (e.g., an $128\times128$ array has 16384 elements), and is difficult to make because of the problems of interconnection of the array elements, cross-talk, thick and stiff connection cable between the transducer and the imaging system. More importantly, such an array has a small clamping capacitance and high electrical impedance that reduce the SNR.

In this paper, both computer simulation and experiment were performed to study the relationship between the quality of images constructed with the Fourier method and the element spacing of array transducers. In the study, two broadband linear array transducers were used. The first is a 48-element array having a center frequency of 2.25 MHz, an aperture of 18.288 mm, an elevation length of 12.192 mm, and an element spacing of 0.381 mm ($0.591\lambda$). The second has 64 elements, a dimension of 38.4 mm $\times$ 10 mm, 2.5 MHz center frequency, and an element spacing of 0.6 mm ($1.034\lambda$). The effective element spacings of these arrays were obtained by combining the received signals of appropriate number of neighboring elements. Results show that resolution of images constructed with the Fourier method is not affected by the element spacing. However, grating lobes of the line spread function of the imaging system increase with the element spacing and the contrast of the images of the cystic objects of an ATS tissue-equivalent phantom decreases dramatically when the element spacing is greater than about $2.365\lambda$. This demonstrates that a 2D array of $2.365\lambda$ element spacing can be used with the Fourier method to construct a reasonably high quality image, which reduces the number of elements from 16384 for a fully sampled ($\lambda/2$ spacing) square 2D array to about 732 that can be manufactured with the current advanced array transducer and interconnection technologies.

## THEORETICAL PRELIMINARY

Assuming that an object is illuminated with a broadband plane wave (pulse), and a flat one-dimensional (1D) or 2D array transducer is used to receive echo signals, from limited diffraction beams such as X waves, one obtains a relationship between the Fourier transform of the backscattering coefficient of biological soft tissues and the received echo signals [1,2]:

$$R_{k_x,k_y,k_z'}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{A(k)T(k)H(k)}{c} F(k_x,k_y,k_z')e^{-i\omega t}dk, \qquad (1)$$

[1]This work was supported in part by the grant HL 60301 from the National Institutes of Health.

  

where $F(k_x, k_y, k_z^{'})$ is a spatial Fourier transform of $f(x, y, z)$ that is an object function representing the distribution of backscattering coefficients of scatterers, $A(k)$ and $T(k)$ are transfer functions of the transmit and receive beams, respectively, $k = \frac{\omega}{c}$ is a wavenumber, $\omega$ is the angular frequency, $c$ is the speed of sound, $t$ is time, $k_z^{'} = k + k_z$, and where $k_x$, $k_y$, and $k_z$ are wavenumbers along the $x$, $y$, and $z$ axes, respectively, and $H(k)$ is the Heaviside step function [10].

If the object is two dimensional, i.e., the object function $f(x, y, z)$ is not a function of $y$, Eq. (1) can be simplified [1,2]:

$$R^{(2D)}_{k_x, k_z^{'}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{A(k)T(k)H(k)}{c} F^{(2D)}(k_x, k_z^{'}) e^{-i\omega t} dk, \qquad (2)$$

where the superscript "$(2D)$" means "two dimensional".

From Eq. (1) or (2), the spatial Fourier transform of the object functions can be obtained from the Fourier transform of the receive signals in terms of time. The image construction method using Eq. (1) or (2) is termed as the Fourier method [1,2].

Because only one transmission is required to construct either a 2D or 3D image, the Fourier method has a potential to achieve a very high frame rate. In addition, because the FFT or IFFT can be used, imaging system can be greatly simplified (no digital delays and multiple-input digital summations are needed as in a conventional digital beamformer).

To construct good images with Eq. (1) or (2), array transducers used should have an element spacing of less or equal to a half of the wavelength concerned. This requires a large number of elements for a given size of an array, especially, for 2D arrays. An array transducer of a large number of elements may have problems as discussed in the Introduction section. However, if the number of element is reduced, quality of constructed images may be lowered. Therefore, it is important to study the trade-off between the quality of constructed images and the number of elements (or the element spacing) of an array.

## SIMULATION

In the simulation, an ATS 539 tissue-equivalent phantom[2] was used [1,2]. The phantom is consist of cylindrical objects of different scattering coefficients relative to the background. The contrasts of these objects are -15 dB, -10 dB, -5 dB, 5 dB, 10 dB, and 15 dB, respectively. In addition to the cylindrical objects, there are line objects in the phantom for testing the resolution of imaging systems. In simulation, the phantom is assumed to have no attenuation.

[2]ATS Laboratories, Inc., Bridgeport, CT.

Two transducers were used in the simulation. One has an aperture of 18.288 mm, 2.25 MHz center frequency, 12.192 mm elevation dimension, element spacing of 0.381 mm ($0.591\lambda$), and 48 total elements. The other is assumed to have a dimension of 38.4 mm $\times$ 10 mm, 2.5 MHz center frequency, 0.6 mm element spacing ($1.034\lambda$) and 64 total elements. Both arrays have no focusing in the elevation direction. The bandwidth of the transducers are assumed to be about 81% of their center frequencies and the speed of sound is assumed to be 1.45 mm/$\mu$s at 23°C in the ATS phantom.

To obtain different element spacings or different number of elements with the arrays described above, received signals from adjacent elements are summed. For example, to increase the element spacing from $0.591\lambda$ to $1.182\lambda$, signals from every two adjacent elements are combined.

Images constructed with the Fourier method and different element spacings are shown on the left hand side of Fig. 1 for both cylindrical objects (see the first 6 columns with contrasts from 15 dB to -15 dB) and a line object (see the 7th and 8th columns). Images in the first 7 columns are constructed with the array of 18.288 mm aperture and images in the 8th column are obtained with the array of 38.4 mm aperture. The element spacings for the images in the first 7 columns are $0.591\lambda$, $1.182\lambda$, $1.774\lambda$, $2.365\lambda$, $3.547\lambda$, and $4.730\lambda$, respectively, from the top to the bottom rows. Notice that the two grey scale bars (upper and lower) on the far right of Fig. 1 have different scales and they are used for the cylindrical and line objects, respectively. It is seen that the -6 $dB$ lateral resolution of the imaging system is almost invariant with the element spacing.

To show the sidelobes and grating lobes of the line spread function (LSF) of the Fourier method, maximum magnitudes of the images in the 7th and the 8th columns of Fig. 1 are plotted versus the lateral distances, and are shown in Fig. 2(1) and 2(2), respectively.

For quantitative study of the contrast of images of the cylindrical objects in Fig. 1, the following formula is used:

$$\text{Contr} = 20\log_{10}\left|\frac{m_i}{m_o}\right|, \qquad (3)$$

where $m_i$ is the mean of the constructed image of a cylindrical object (see the circles in Fig. 1), $m_o$ is the mean of the background of the phantom, and "Contr" is an image contrast that represents the ratio of the means inside and outside of the cylindrical objects in dB scale.

The image contrast (normalized to $\pm 1$) versus element spacing is shown in Fig. 3(1). It is seen that the image contrast decreases as element spacing increases.

## EXPERIMENT

The experiment was performed with the same transducer specifications and the ATS phantom as those used in the simulation except that the phantom has a frequency-dependent attenuation of 0.5 dB/cm/MHz, the width of transducer elements is not zero, and the bandwidths of the transducers are about 40% of their center frequencies [2]. The attenuation was compensated with a time-gain control (TGC) in image constructions (see the blockdiagram of the data acquisition of the Fourier method in Fig. 3 of Reference [2]). As in the simulation, to obtain a larger effective element spacing, received signals from adjacent elements are summed.

Experiment results are shown in the right hand side panels of Fig. 1 (from columns 9 to 16). These panels correspond to those obtained with the simulation except in columns 15 and 16, images of more than one line object are constructed. In addition, in column 16, there are only 3, instead of 6, images. This is because from a 64 element array, summation of signals of adjacent elements produce only arrays of effective number of elements of 32, 16, or smaller, corresponding to an element spacing of $2.069\lambda$, $4.138\lambda, \cdots$, respectively.

The contrast of the constructed images of cylindrical objects is shown in Figs. 3(2). It is seen that they are less sensitive to the element spacing than those obtained with the simulation. The is because elements of any practical array have a certain width and the directivity patter of each element is not cylindrical (or omni-directional). This means that in practice, an array of a larger element spacing or a smaller number of elements can be used with the Fourier method.

## Discussion

In previous sections, we have seen the results of the simulation and the experiment using 1D ultrasonic linear array transducers for image constructions with the Fourier method. The images in Fig. 1 demonstrate that good images can be constructed with the Fourier method even though the element spacing is increased greatly or the number of elements is reduced dramatically. Figs. 3(2) gives a similar result showing image quality is deteriorated greatly only after the element spacing is larger than about $2.365\lambda$. In addition, Fig. 1 shows that grating lobes of the line spread function of the Fourier method do not affect significantly the image contrast of the cylindrical objects if the element spacing is smaller than $2.365\lambda$.

The resolution of images constructed with the Fourier method is almost constant over a wide range of element spacing. This means that given an array aperture, the image resolution is almost constant (Figs. 1 and 2).

## Conclusion

A Fourier method is recently developed that can be used to construct images at a high frame or volume rate (about 3750 frames/s at a depth of about 200 mm in biological soft tissues), in addition to many other advantages of the new method [1-3]. In this paper, computer simulation and experiment were performed to study the relationship between the quality of images constructed with the Fourier method and the element spacing of array transducers. Results show that array transducers of element spacing as large as $2.365\lambda$ can be used to construct images of good quality. This reduces the number of elements of a fully sampled $128 \times 128$ 2D array from 16384 to about 732 and thus imaging systems with the Fourier method can be greatly simplified.

## References


[1] Jian-yu Lu, "2D and 3D high frame rate imaging with limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 44, no. 4, pp. 839-856, July, 1997.

[2] Jian-yu Lu, "Experimental study of high frame rate imaging with limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 45, no. 1, pp. 84-97, January, 1998.

[3] Jian-yu Lu, "Transmit-receive dynamic focusing with limited diffraction beams," in *IEEE 1997 Ultrasonics Symposium Proceedings*, 97CH36118, vol. 2, pp. 1543-1546, 1997.

[4] Jian-yu Lu and J. F. Greenleaf, "Nondiffracting X waves — exact solutions to free-space scalar wave equation and their finite aperture realizations," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 39, no. 1, pp. 19-31, January, 1992.

[5] Jian-yu Lu and J. F. Greenleaf, "Experimental verification of nondiffracting X waves," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 39, no. 3, pp. 441-446, May, 1992.

[6] Jian-yu Lu, Hehong Zou, and J. F. Greenleaf, "Biomedical ultrasound beam forming," *Ultrasound in Medicine and Biology*, vol. 20, no. 5, pp. 403-428, July, 1994.

[7] Jian-yu Lu, "Limited diffraction array beams," *International Journal of Imaging System and Technology*, vol. 8, no. 1, pp. 126-136, January, 1997.

[8] Jian-yu Lu, "Designing limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 44, no. 1, pp. 181-193, January, 1997.

[9] Jian-yu Lu, "Bowtie limited diffraction beams for low-sidelobe and large depth of field imaging," *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.*, vol. 43, no. 6, pp. 1050-1063, 1995.

[10] R. Bracewell, *The Fourier Transform and its Applications.* New York, NY: McGraw-Hill Book Company, 1965, chs. 4 and 6.

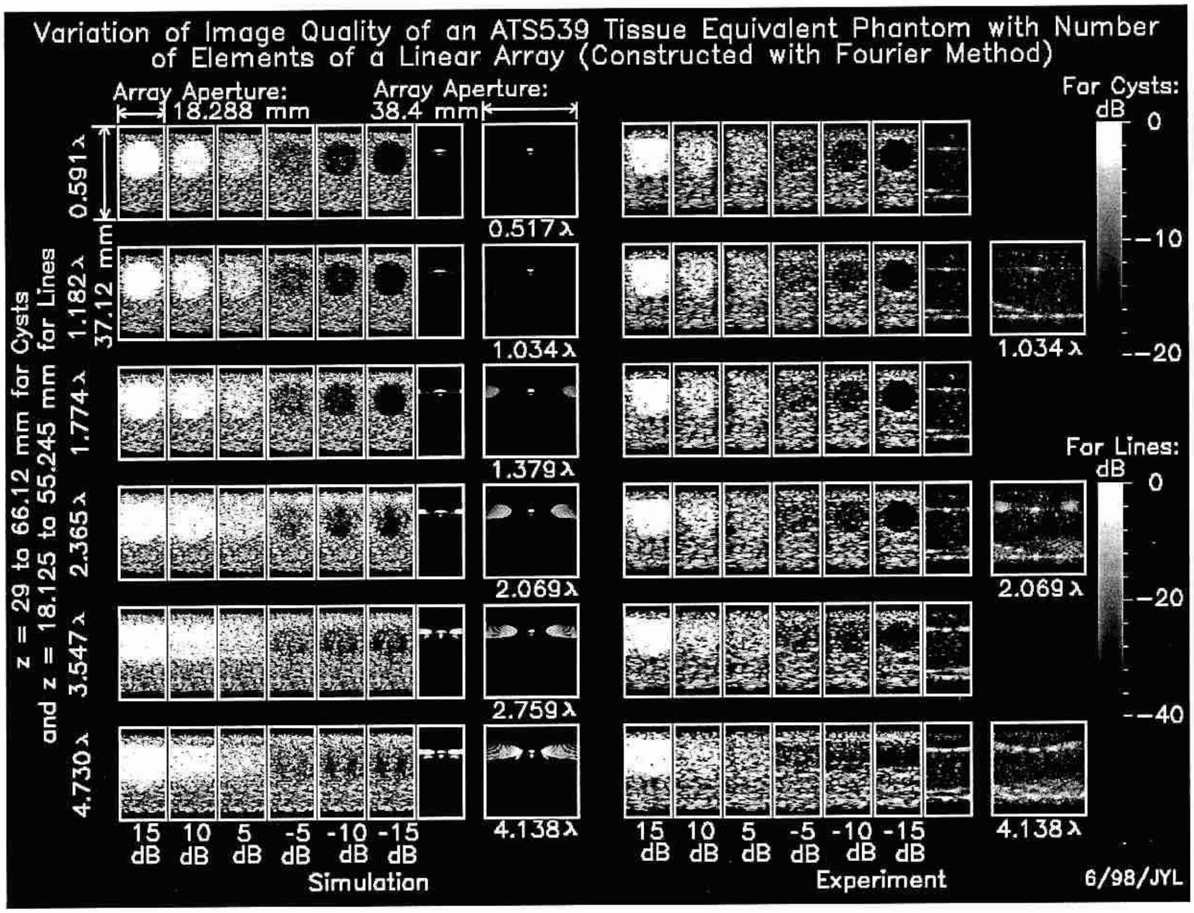


Figure 1: Images of objects of an ATS phantom constructed with the Fourier method and with different element spacings.

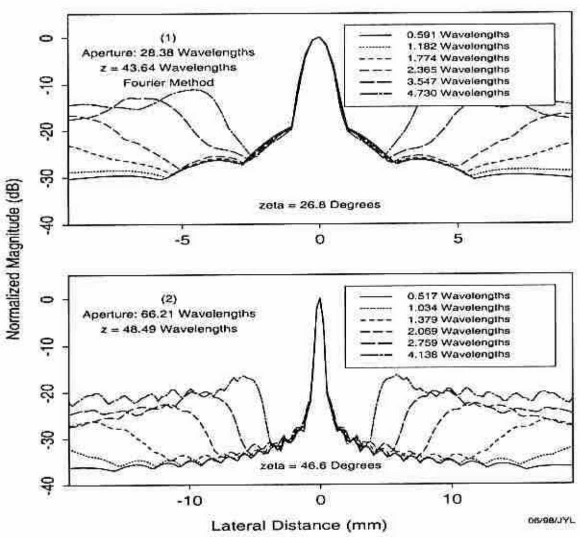


Figure 2: Sidelobes and grating lobes of the Fourier method. (1) 18.288 mm aperture. (2) 38.4 mm aperture.

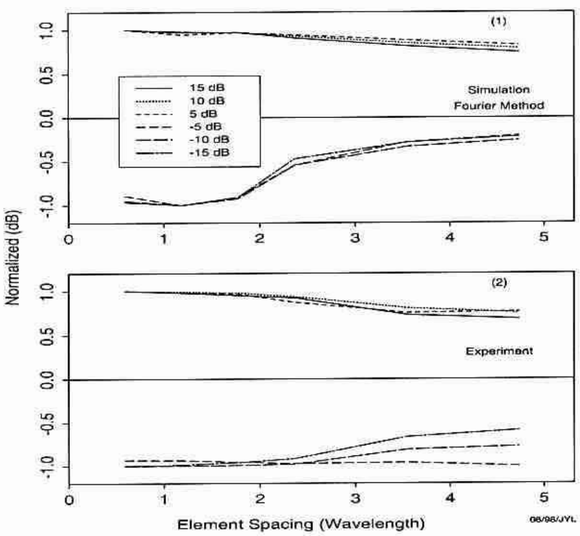


Figure 3: Normalized contrast as a function of the element spacing. (1) Simulation. (2) Experiment.