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Optical X wave communications

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Abstract

This paper presents a new communication system using optical X waves which, in theory, can propagate to an infinite distance without spreading. In practice, when these waves are produced with a finite aperture and energy, they have a large depth of field. A system model using optical X waves as carriers to transfer signals is established. The relationship between optical X wave and the conventional communication systems is studied. Simulation demonstrating the performance of the optical X wave communication systems is presented.

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1. Introduction

Recently, there is a great interest in the study of X waves for medical imaging and other physics related areas [1–11]. Theoretically, X waves can propagate to an infinite distance without changing their shapes if they are produced with an infinite aperture and energy [2]. In practice, when these waves are produced with a finite aperture and energy, they have a large depth of field [3, 4]. Because of this property, they can have applications in communication systems. Using optical X waves in communications has several advantages. First, it can increase the capacity of the communications because optical X waves are spatially orthogonal which allows a multiple-ring to emit. Second, it is secure because the waves can only be detected by an optical X wave array that has the same array response as that of transmission. Third, it has a low probability of interception due to the precise pointing and tracking provided by the narrow transmitting beams with little diffraction. Finally, it may have less interference to other sources of communications and has less multi-path effects because optical X waves propagate in a confined space and the transmitter and the receiver have to be collimated with each other.

Applications of optical X wave communications may span areas in commercial and military markets as well as scientific research. As the need for personal communication services expand, optical X wave systems will become increasingly important to multi-node commercial communication networks. In all these instances, optical X wave technology can make use of current radio frequency (RF) technologies by receiving RF or optical X wave communications from the ground, transferring the information with optical X wave through space, and retransmitting via RF or optical X wave carriers to a destination ground site. The optical X wave system can also be operated in parallel with other assets, since it is spatially confined and collimated. Optimization of current and future space and ground assets will provide rapid global transfer of digital images, voice and other data in future secure high-speed communication networks.

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2. X wave concept

An n-dimensional scalar wave equation for source-free, lossless, and isotropic/homogeneous media is given by [2]

$$\left(\sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) \Phi = 0 \qquad (1)$$

where x_j $(j=1,2,\dots,n)$, represent rectangular coordinates in an N-dimensional space, t is time, n is an integer, c is a constant and represents the speed of the wave, and $\Phi = \Phi(x_1, x_2, \dots, x_n; t)$ is an n-dimensional complex wave field.

Letting $n=3$, $x_1 = x$, $x_2 = y$, $x_3 = z$, one obtains an mth-order non-rotating X wave (as opposed to the rotating X waves given by Eq. (12) in Ref. [2])

$$\Phi_{X_m}(\boldsymbol{r},t) = \cos m(\phi - \phi_0) \int_0^{\infty} B(k) J_m(kr\sin\zeta) \times \mathrm{e}^{-k[a_0 - i\cos\zeta(z - c_1 t)]}\,\mathrm{d}k, \qquad (2)$$

where m is an integer, $m = 0,1,2,\dots$, J_m is an mth-order Bessel function of the first kind, $\boldsymbol{r} = (r,\phi,z)$ is a point in space, $\phi = \tan^{-1}(y/x)$ is the azimuthal angle, and ϕ_0 is the initial azimuthal angle (polarization) of the beams at the plane $z=0$, $B(k)$ is the transmitting or receiving transfer function of a microwave antenna, an optical device, or an acoustic transducer, a_0 is the constant that determines the fall off speed of the high frequency component of the X waves, ζ is the Axicon angle [2], and $c_1 = c/\cos\zeta$ is the phase velocity. Eq. (2) is also an exact solution to Eq. (1). From Eq. (2), it can be seen that X waves have a constant phase velocity for all frequency components or have an infinite depth of field. In practice, the aperture of a wave source is always finite. In this case, X waves have a finite but large depth of field, i.e., they can propagate to a large distance without significant distortions.

3. System mode

The block diagram of the optical X wave communication system with a ring antenna is shown in Fig. 1, where the signal transmitted by the ith ring transmitter is given by [1,12]

$$S_T(r_i', t) = \sqrt{2P_{T_i}}\, d_i(t) \left[f_i^{\mathrm{I}}(t) \cos\omega_0 t + f_i^{\mathrm{Q}}(t) \sin\omega_0 t \right], \qquad (3)$$

where r_i' is the radius of the ith ring of the transmitter, $i = 1,2,\dots,N$, P_{T_i} is the power amplitude of the ith transmitted signal, ω_0 is the angular carrier frequency, $d_i(t)$ is the binary data of the ith information, $f_i^{\mathrm{I}}(t)$ and $f_i^{\mathrm{Q}}(t)$ are in-phase and quadrature weights of the format of the arbitrary signals (modulation format), respectively.

Immediately after the transmitting lens (rings are placed at the focal plane of the lens), the signal is carried by an optical X wave that is proportional to the spatial Fourier transform of a ring [1],

$$\Phi_{X_0}^{i}(\boldsymbol{r},t) \propto \mathscr{F}_s[S_T(r_i',t)], \qquad (4)$$

where $\Phi_{X_0}^{i}(\boldsymbol{r},t)$ is a zeroth-order optical X wave produced by the ith ring. (For simplicity, we consider only the zeroth-order X waves. Higher-order X waves require rings modulated by $\cos m(\phi - \phi_0)$, where ϕ_0 is the initial azimuthal angle, see Eq. (2), and is not rotary symmetric.) $\mathscr{F}_s$ is the spatial Fourier transform. If the Axicon angle of the optical X waves (see Eq. (2)) is zero (the ring diameter is zero and the ring is shrunk to a point source), the wave-field after the lens is a plane wave and is not a

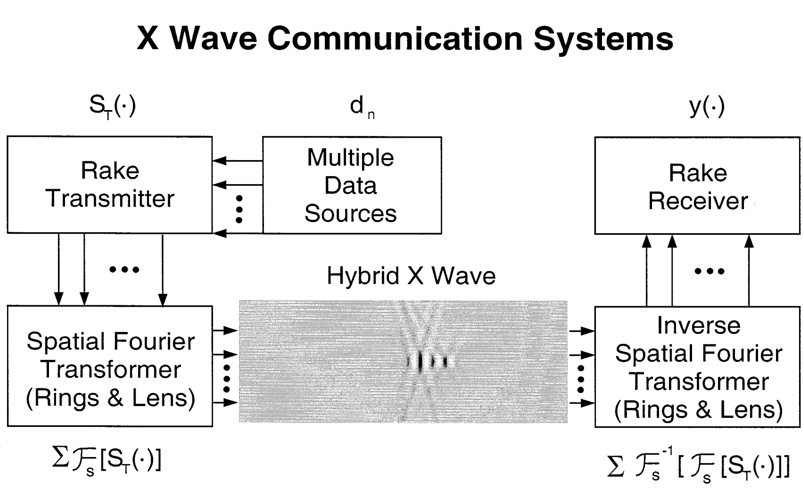

Fig. 1. Model of the optical X wave communication systems.

function of $\boldsymbol{r}$ and the optical X wave communication system becomes a conventional system.

If multiple optical X waves are produced simultaneously by multiple rings, a hybrid optical X wave, $\Phi_{X_0}(\boldsymbol{r},t)$, that is a linear superposition of all the individual optical X waves is produced. In this case, data are naturally scrambled at the passage of the waves. The hybrid optical X wave is given by

$$\Phi_{X_0}(\boldsymbol{r},t)=\sum_{i=1}^{N}\Phi^{i}_{X_0}(\boldsymbol{r},t). \tag{5}$$

At the receiver, a device (a lens in optics and acoustic cases) is used to separate the optical X waves and decomposes them back into rings (Fig. 1). At the focal distance after the lens of the receiver, the received signal at the ith ring is proportional to the inverse spatial Fourier transform of the optical X wave produced with the ith transmitting ring

$$\begin{aligned}S_R(r_i,t)&\propto\mathscr{F}_s^{-1}\left[\Phi^{i'}_{X_0}(\boldsymbol{r},t)+N_i(t)\right]\\&=T_x(t)*\left[S_T(r_i',t-t_0)+n_i(t)\right],\end{aligned} \tag{6}$$

where $\Phi^{i'}_{X_0}(\boldsymbol{r},t)$ is approximately a delayed version of $\Phi^{i}_{X_0}(\boldsymbol{r},t)$, "$*$" is the convolution with respect to time, $T_x(t)$ is the impulse response of the optical X wave communication system, which is related to the aperture size and the propagation distance of the optical X wave (With an infinite aperture, $\mathscr{F}[T_x(t)]\equiv 1$. For simplicity, in the following, we assume $\mathscr{F}[T_x(t)]\equiv 1$.) $N_i(t)$ is the Fourier transform of the noise of the ith channel, $n_i(t)$, that is assumed to be the white Gaussian noise, and t_0 is the delay time.

Following the conventional detection method, the output of the ith ring is given by

$$y^{i}_{\mathrm{ring}}(t)=\int_{-\infty}^{\infty}S_R(r_i,t-\tau)R^{i}_{\mathrm{ring}}(\tau)\,\mathrm{d}\tau, \tag{7}$$

where r_i is the radius of the ith ring and $R^{i}_{\mathrm{ring}}(t)$ can be expressed as

$$R^{i}_{\mathrm{ring}}(t+t_0)=\sqrt{2P_{R_i}}\left[f_i^{\mathrm{I}}(t)\cos\omega_0(t)+f_i^{\mathrm{Q}}(t)\sin\omega_0(t)\right]. \tag{8}$$

Note that $S_R(r_i,t)$ here is not a function of $\boldsymbol{r}$ and $y^{i}_{\mathrm{ring}}(t)$ includes the transfer function of the receiver ring (for simplicity, we also assume that the transfer functions of receiver rings are equal to 1).

The above convolution is expressed by

$$y^{i}_{\mathrm{ring}}(t)=R^{i}_{X_TX_R}(t)+R^{i}_{n_TX_R}(t), \tag{9}$$

where

$$\begin{aligned}R^{i}_{X_TX_R}(t)=2\sqrt{P_{T_i}P_{R_i}}\int_{-\infty}^{\infty}&d(t-t_0-\tau)\\&\times\left[f_i^{\mathrm{I}}(t-t_0-\tau)\cos\omega_0(t-t_0-\tau)\right.\\&\left.+f_i^{\mathrm{Q}}(t-t_0-\tau)\sin\omega_0(t-t_0-\tau)\right]\\&\times\left[f_i^{\mathrm{I}}(\tau-t_0)\cos\omega_0(\tau-t_0)\right.\\&\left.+f_i^{\mathrm{Q}}(\tau-t_0)\sin\omega_0(\tau-t_0)\right]\mathrm{d}\tau.\end{aligned} \tag{10}$$

After filtering, synchronous detection and sampling, the convolution of the desired signals from each ring is given by

$$R^{i}_{X_TX_R}(T)=\tfrac{1}{2}\sqrt{P_{T_i}P_{R_i}}\int_0^T d(t)\left[\left(f_i^{\mathrm{I}}(t)\right)^2+\left(f_i^{\mathrm{Q}}(t)\right)^2\right]\mathrm{d}t, \tag{11}$$

where T is the signal cycle.

The energy content of the received signals is as follows,

$$\int_0^T\left[\left(f_i^{\mathrm{I}}(t)\right)^2\right]\mathrm{d}t=C_IT \tag{12}$$

and

$$\int_0^T\left[\left(f_i^{\mathrm{Q}}(t)\right)^2\right]\mathrm{d}t=C_QT \tag{13}$$

for the in-phase and quadrature components, respectively, and where C_I and C_Q are in-phase and quadrature constants, respectively. The demodulated signals are given by

$$R^{i}_{X_TX_R}(T)=\pm\sqrt{P_{T_i}P_{R_i}}\,CTd_n, \tag{14}$$

where $C=\frac{1}{2}(C_I+C_Q)$ and d_n is the binary data of the information.

As in the conventional analysis [12], the convolution of the noise at each receiver ring is written as

$$R^{i}_{n_TX_R}(t)=\sqrt{2P_{R_i}}\int_{-\infty}^{\infty}n^i(t-\tau)\left[f_i^{\mathrm{I}}(\tau)\cos\omega_0\tau+f_i^{\mathrm{Q}}(\tau)\sin\omega_0\tau\right]\mathrm{d}\tau. \tag{15}$$

After filtering, synchronous detection and sampling, the convolution of the noise at each ring (channel) is given by [12]

$$R^{i}_{n_TX_R}(T)=L_I+L_Q, \tag{16}$$

where L_I and L_Q are in-phase and quadrature components of the convolution of the noise, respectively. Because the noise and the array response are uncorrelated, the effects of the noise component are very small.

The input to the decision device is

$$L=y^{i}_{\mathrm{ring}}(T)=\pm Ad_n+L_I+L_Q, \tag{17}$$

where A is a constant.

If $d_n = +1$ represents the logic symbol 1 and $d_n = -1$ represents the logic symbol 0, the decision device produces the symbol 1 if $L > 0$ and the symbol 0 if $L < 0$. An error occurs if $L < 0$ when $d_n = +1$ or if $L > 0$ when $d_n = -1$. The probability that $L = 0$ is zero.

4. Simulation results

Simulation of optical X wave communications is performed for a laser (optical) communication system and is based on the optical X wave theory [1,2,4,8]. Zeroth-order optical X waves (axially symmetric) are assumed in the simulation. A coaxial multi-ring laser gun is used as a wave source. The rings of a laser gun are modulated individually to form different signal channels. Assuming there are two rings (channels) and the outputs of the rings are shrunk with an optical imaging system to form two smaller rings of radii of about 10 and 20 μm, respectively, the transmitting system in Fig. 1 can be used to produce multiple optical X waves directly and transfer signals through the space over a large distance in parallel.

The results of the simulation are shown in Fig. 2. The conditions for the simulation are given in the figure (wavelength is assumed to be 1.55 μm (center frequency of about 193548.4 GHz) and both the diameter and the focal length of the transmission lens are 50 mm). With these parameters, the depths of field of the optical X waves [2] are about 125 and 62.5 m for the rings of 10 and 20 μm, respectively, corresponding to Axicon angles of about 0.01146° and 0.02292°. The bandwidth of the optical X wave produced is determined by that of the excitation signal and the transfer functions of the ring radiators. The simulation was performed for both broadband (1.5-cycle, see images in the left column of Fig. 2) and narrower-band signals (10-cycle tone burst, see images in the right column). In microwave and acoustic cases, short pulses can be produced. In optics, tone bursts that are much longer than 10 cycles are usually used. The hybrid optical X waves produced with both the 10 and 20 μm radius rings immediately after the transmitting lens are shown in the top row of Fig. 2. Waves after propagation over 60 m are shown in the middle row, and waves at the detecting rings are shown in the bottom row. Notice that because the optical X waves produced by the 20 μm radius ring have a slightly higher phase velocity (larger Axicon angle), they are advanced in time at the receiver site [2]. The superluminal nature of X waves has been discussed elsewhere [2,4,7].

From the optical X waves shown in Fig. 2(e) and 2(f), it is seen that ring fields are well recovered at the focal

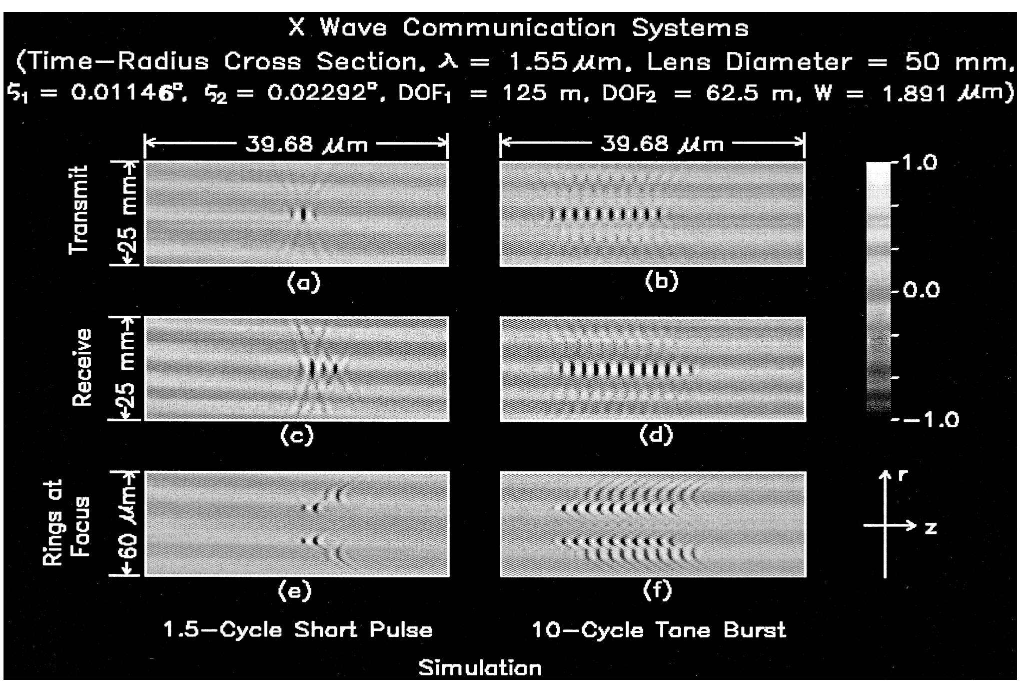

Fig. 2. Simulation of optical X wave communication systems. (a), (b) Hybrid optical X waves produced with both rings of 10 and 20 μm radii. (c), (d) Hybrid optical X waves before the reception lens after propagating over 60 m. (e), (f) Hybrid optical X waves at the focal plane of the reception lens. In (a), (b), (c) and (d), the simulation is carried out with the Fresnel approximation, while in (e) and (f), with the Rayleigh-Sommerfeld diffraction formula. Images in the left and right columns correspond to broadband (1.5 cycles) and narrower band (10 cycles) results, respectively.

plane of the receiver lens. Ring detectors are placed at the peaks of the ring fields corresponding to the transmitting rings in the transmitter (Fig. 1) to recover the binary signals of multiple channels simultaneously. From Fig. 2(e) and 2(f), it is clear that more transmitting rings and detectors can be added to increase the number of channels of the optical X wave communication system, including to add a center point transmitter and receiver for conventional communications.

Assuming that the optical X wave transmitter transmits uncorrelated symbols with a linear modulation, e.g., quadrature phase-shift keying (QPSK), the information that is carried by the optical X waves from both channels can travel a minimum distance of 62.5 m, without significant distortions.

5. Discussion

5.1. Potentials of the X wave communication systems

The high carrier frequency and bandwidth potential of the optical X wave communication systems make them attractive for widespread use in future communication systems. Using an X wave communication system, X waves carrying digital and analog information can support real-time interconnections between any two points on the globe at data rates of several gigabits per second or higher. This data transfer can be realized using X wave array transmitters and receivers with lower power requirements (because the X wave propagation energy concentrates in one direction) than used in current radio frequency (RF) systems operating at lower data rates. X waves generated by a finite aperture are of practical importance due to three aspects when applied to the communication systems as discussed in the Introduction: high speed, secure, and less sensitive to multiple-path effects. Therefore, the performance of communication systems can be improved enormously by using X waves.

5.2. Other orthogonal modes

In addition to using the Axicon angles (the example shown in the simulation results), other parameters of X waves could also be used in combination with the Axicon angles to further increase the capacity of X wave transmission systems. For example, the parameter, m, in Eq. (2) may be used to separate channels because X waves of different order, m, are also orthogonal (for $m>0$, X waves are not rotary symmetric [2]). The advantage of using asymmetrical X waves is that they all have the same depth of field as the zeroth-order X wave when the Axicon angle is the same. Similarly, the initial phase, ϕ_0, in Eq. (2) could also be used to distinguish channels in combination with the parameters, m and ζ. Of course, methods that may increase data rate of conventional communication systems are also applicable to each channel of the X wave communication systems.

5.3. Array beams to further increase the number of channels

It is worth noting that array beams [11] could also be used in the communication system. In this case, point sources and detectors are used to replace rings. This will greatly increase the number of channels. However, the signal-to-noise ratio may be reduced because the transmitter and detector sizes will be small and the detectors will be phase insensitive.

With a rotating X wave [2,8] multiplied by $i^n e^{-im\theta}$ and summing over the order of the wave, m, broadband limited diffraction array beams are obtained that are also limited diffraction solutions to the isotropic homogeneous scalar wave equation [11]:

$$\Phi_{\text{Array}}(\boldsymbol{r},t)=(\cos k_x x)(\cos k_y y)\,e^{ik_z(z-c_1 t)}, \tag{18}$$

where $k_z=\sqrt{k^2-\left(k_x^2+k_y^2\right)}$, and where k_x and k_y are wave numbers in the x and y directions, respectively. Point sources that are placed at or away from the wave axis can be used to produce grid array beams of different combinations of k_x and k_y.

5.4. Alignment of the X wave systems

In Fig. 2, the ring detectors used are phase sensitive. This requires that the transmitting rings, lenses, and the ring detectors must be coaxially aligned with a precision of a fraction of a wavelength and perpendicular to the beam axis to avoid phase cancellation. Because of the phase sensitivity, background noise from random radiation sources will be largely cancelled and the system will have a high security (interception without disrupting the normal communications of such systems is difficult).

5.5. Trade offs

From Fig. 2, it is seen that the ring fields reconstructed from the X waves have a certain width that increases with the decrease of the diameter of the reception lens. In addition, sidelobes of the ring fields will increase the effective ring width. A larger width will reduce the number of rings that can be placed in the system. The number of rings will also be reduced if the communication distance is increased. This is because a larger distance decreases the largest usable diameter of the rings.

It is well known that the radius of the Airy pattern of a circular wave source is given by [8]

$$w=0.61\lambda F/a_r, \tag{19}$$

where λ is the wavelength, F is the focal length, and a_r is the radius of the lens aperture of the receiver.

With the X wave communication system discussed in this paper, the relationship between the Axicon angle and the radius of the ring can be obtained [8],

$$\zeta = \sin^{-1}(a_{\rm ring}/F), \tag{20}$$

where $a_{\rm ring}$ is the radius of the transmitting rings.

The maximum propagation distance without significant signal distortion is given by (depth of field) [2]

$$Z_{\max} = a_t \cot\zeta. \tag{21}$$

where a_t is the radius of the lens of the transmitter.

With a wavelength of 1.55 μm and $a_r = 25$ mm, the radius of the Airy pattern is about 1.891 μm. This pattern will be spatially convolved with any ring responses from the X waves, i.e., it will increase the widths of rings at the receiver because it is multiplied with the aperture of the incident X waves. Using the example of this paper, the maximum number of rings that can be placed within the 20 μm radius is about 10. Because the incident X waves in front of the receiver have some distortions from those immediately after the transmission lens, the actual number of rings may be smaller (see Fig. 2).

When the communication distance is determined, the Axicon angle of an X wave can be calculated with Eq. (21). Then, from Eq. (20), the maximum radius of the ring can be calculated. With the radius of the Airy pattern (Eq. (19)), the maximum number of rings that can be placed in an X wave communication system can be estimated.

From the discussion above, it is clear that given the wavelength of the carrier and the *f*-number (F/D, where D is the diameter of the aperture) of the transmitter and receiver, the communication distance is inversely proportional to the maximum number of channels or the data rate of the X wave communication systems.

5.6. Microwave communications

The current method could also be extended to a microwave communication system. Because a microwave wavelength is much larger than that of an optical wave, a large aperture is required to achieve a large communication distance (Eqs. (19)–(21)). If the center frequency is 200 and 700 GHz, and the aperture diameters are 10 and 20 m, the radii of the Airy patterns of these systems will be about 3.66 and 0.5229 mm, respectively. For the first system, if *f*-number = 1 and there are two transmission rings of radii of 5 and 10 mm (corresponding Axicon angles of 0.0286° and 0.0573°), respectively, the maximum communication distance will be 5 km (the maximum number of rings is about 2.73). With the second system, the maximum communication distance is 115 km if the Axicon angle is 0.005° (a ring radius of 1.745 mm) and *f*-number = 1. The maximum number of rings is about 3.34. To construct a microwave X wave communication system, ring antennas with proper focusing mechanisms must be developed.

6. Conclusion

In this paper, a new communication system with limited diffraction beams – optical X waves for parallel transmission of digital binary signals has been developed. An example has been given to demonstrate the method. The use of optical X waves for communication systems has several advantages over conventional systems, such as larger capacity of communications, smaller multi-path effects, and high security features. The theoretical analysis and simulation results show that optical X waves can significantly improve the performance of communication systems.

Acknowledgements

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