ACTA PHYSICA SINICA

(Overseas Edition)

PRINCIPLE AND APPLICATIONS OF LIMITED DIFFRACTION BEAMS

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Limited diffraction beams are new beamforming techniques developed recently. These beams have a large depth of field and have potential applications in medical imaging, tissue identification, nondestructive evaluation of industrial materials, Doppler velocity measurement, high-speed private communications, as well as other physics—related areas such as electromagnetics and optics. This paper presents the basic principles of these novel beams and their major applications.

PACC: 4320; 4385

I. INTRODUCTION

Limited diffraction beams were first discovered by Stratton in 1941.[1] These beams can propagate to an infinite distance without spreading provided they are produced with an infinite aperture and energy. Even with a finite aperture and energy, they have a large depth of field. Because of this property, limited diffraction beams have been studied by many investigators in medical imaging[2-7], tissue identification[8], nondestructive evaluation (NDE) of industrial materials[9], Doppler velocity vector measurement[10,11], secure high-speed communications[12], as well as other physics-related areas such as electromagnetics[13,14] and optics.[15-17] Detailed properties of these beams have also been studied.[18-40]

In this paper, the basic principles of limited diffraction beams are briefly reviewed and their major applications are presented.

II. BASIC PRINCIPLES

A source-free loss-less n-dimensional isotropic/homogeneous scalar wave equation in rectangular coordinates is given by[41]

$$\left[\sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right] \Phi = 0, \tag{1}$$

where $x_j (j = 1, 2, \cdots, n)$, represent rectangular coordinates in n-dimensional space, t is time, n is an integer, c is a constant. and $\Phi = \Phi(x_1, x_2, \cdots, x_n; t)$ is an n-dimensional wave field.

One of the special families of solutions of the n-dimensional wave equation (Eq. (1)) is given by[39,4]

$$\Phi(x_1, x_2, \cdots, x_n; t) = f(s). \tag{2}$$

where

$$s = \sum_{j=1}^{n} D_j x_j - Et \qquad (n \geq 1), \tag{3}$$

and where

$$E = \pm c \sqrt{\sum_{j=1}^{n} D_j^2} \qquad (n \geqslant 1). \tag{4}$$

The D_j are complex coefficients which could relate to the jth component of an n-dimensional vector wave number and are independent of the spatial and time variables (x_j ($j = 1, 2, \cdots, n$) and t), and $f(s)$ is any complex function (well-behaved) of s.

If $n = 0$, $f(s)$ is only a function of time t, and will not represent a wave. Therefore, we always assume that $n \neq 0$.

Equation (3) can be rewritten as

$$s = \sum_{j=1}^{n-1} D_j x_j + D_n(x_n - c_1 t) \qquad (n \geqslant 1), \tag{5}$$

where

$$c_1 = E/D_n = \pm c \sqrt{1 + \sum_{j=1}^{n-1} D_j^2/D_n^2} \qquad (n \geqslant 1). \tag{6}$$

If c_1 in Eq. (5) is real, $f(s)$ represents a limited diffraction wave propagating along axis, x_n, at the phase velocity of c_1, in an n-dimensional space, i. e., traveling with the wave, one sees a constant complex wave pattern. "$\pm$" term in Eq. (6) represents forward and backward propagating waves, respectively. In the following, we consider forward going waves only. For backward going waves, the results are similar.

In the following, we derive a limited diffraction beam (X wave[39]) as an example in a three-dimensional space.

If $n = 3$, $x_1 = x$, $x_2 = y$, and $x_3 = z$, Eq. (1) is a three-dimensional wave equation. Assume that $D_1 = a_0(k, \zeta)\cos\theta$, $D_2 = -a_0(k, \zeta)\sin\theta$, and $D_3 = b(k, \zeta)$, where θ, k and ζ are free parameters which are independent of the spatial position, $r = (x, y, z)$, and time, t, and $a_0(k, \zeta)$ and $b(k, \zeta)$ are any well-behaved complex functions of k and ζ. From Eq. (5), we obtain[39]

$$s = a_0(k, \zeta)x\cos\theta - a_0(k, \zeta)y\sin\theta + b(k, \zeta)(z - c_1 t), \tag{7}$$

where

$$c_1 = c\sqrt{1 + a_0^2(k, \zeta)/b^2(k, \zeta)}. \tag{8}$$

Integrating $f(s)$ (see Eq. (2)) over free parameters k and θ, one obtains[39]

$$\Phi_\zeta(s) = \int_0^\infty T(k)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} A(\theta)f(s)\mathrm{d}\theta\right]\mathrm{d}k, \tag{9}$$

where $T(k)$ is any well-behaved complex function of k, and $A(\theta)$ represents any well-behaved complex weighting function of the integration with respect to θ.

Letting $a_0(k, \zeta) = -\mathrm{i}k\sin\zeta$, $b(k, \zeta) = \mathrm{i}k\cos\zeta$, $f(s) = \mathrm{e}^s$, $A(\theta) = \mathrm{i}^m\mathrm{e}^{\mathrm{i}m\theta}$, and $T(k) = B(k)\mathrm{e}^{-a_0 k}$, and using an integral representation of the Bessel functions[42], we obtain from Eq. (9) an mth-order X wave solution[39],

$$\Phi_{X_m} = \mathrm{e}^{\mathrm{i}m\phi}\int_0^\infty B(k)\mathrm{J}_m(kr\sin\zeta)\mathrm{e}^{-k[a_0 - \mathrm{i}\cos\zeta(z - c_1 t)]}\,\mathrm{d}k \qquad (m = 0, 1, 2, \cdots), \tag{10}$$

which is the Laplace transform of the function, $B(k)\mathrm{J}_m(kr\sin\zeta)$, where $B(k)$ is any well-behaved complex function of k and is a temporal frequency transfer function of a radiator

system, J_m is the mth-order Bessel function of the first kind, $c_1 = c/\cos\zeta$ is the phase speed of the X waves, $a_0 > 0$ is a constant, and $r = \sqrt{x^2 + y^2}$ and ϕ are variables of polar coordinates.

If $m = 0$ and $B(k) = a_0$, one obtains a broadband zeroth-order X wave[39],

$$\Phi_{\mathrm{XBB}_0} = \frac{a_0}{\sqrt{(r\sin\zeta)^2 + [a_0 - i(z\cos\zeta - ct)]^2}}, \tag{11}$$

where the subscript "XBB" means "broadband X wave".

III. APPLICATIONS

Because limited diffraction beams such as the X waves have a large depth of field, they have many applications.

A. Medical imaging

Conventional medical imaging uses focused beams. These beams have a short depth of field, i. e., they have a small beamwidth only at the focal depth. With limited diffraction beams, high resolution images can be obtained over a large depth of field without using montage that reduces image frame rate. [2-7]

Limited diffraction beams can also be used to obtain high frame rate images (up to 3750 frames or volumes/second in biological soft tissues at a depth of about 200 mm) with simple electronics. [21,22] This is because images can be constructed with the fast Fourier transform (FFT) and the inverse faster Fourier transform (IFFT).

In addition, limited diffraction beams can be used to construct images of quality comparable to a two-way (transmit-receive) dynamic focusing imaging system[20] without using montage. This maintains a high frame rate (no need of montage).

B. Tissue identification

Limited diffraction beams have a large depth of field. This may simplify tissue identification because no diffraction corrections for beams are necessary. [8]

C. Nondestructive evaluation of industrial materials

The speed of sound of industrial materials varies in a wide range. With conventional focused beam, the focal distance changes with the materials of different speed of sound. Limited diffraction beams focus right from the surface of transducer up to the entire region of interest. This may simplify material characterizations since no re-focusing is needed. [9]

D. Secure high speed communications

Because of limited diffraction beams of different parameters such as different Axicon angle[39] may be orthogonal with each other. Therefore, multiple limited diffraction beams can be superposed to form a hybrid beam that is transmitted through the same communication channel. At the receiver, different limited diffraction beams can be recovered simultaneously. If each limited diffraction beam is used as a carrier of each channel of signal, the communication capacity will be increased greatly. [12] In addition, since limited diffraction beams are

collimated, interception of signals by intruders will be difficult, i.e., the communications are more secure.

E. Doppler velocity vector measurement

Conventional Doppler imaging system only measures blood flow velocity components in the direction of beams. Limited diffraction beams such as Bessel beams[15-17], grid array beams, and layered array beams[23,24] can be used to measure both the transverse and axial components of the blood flow velocity accurately.[10,11] This is possible because limited diffraction beams have distinct spatial beam patterns that modulate Doppler signals produced by moving objects. From both the transverse and axial velocity components, velocity vector can be calculated.[10,11]

F. Electromagnetics

Because of the large depth of field, limited diffraction beams can be used as a guiding system to deliver warheads to target or used as electromagnetic bullets directly.[13,14]

G. Optics

Recently, X waves have been produced with a laser beam.[43] In addition, limited diffraction beams are applied to optical lithography for etching of integrated circuits. A higher resolution and larger depth of field are reported.

H. Physics

Limited diffraction beams have been extended to quantum mechanics and other physics areas.[44]

ACKNOWLEDGMENT

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