Sidelobe reduction of limited diffraction beams with Chebyshev aperture apodization

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A limited diffraction beams (LDB) imaging system with Chebyshev weighting is presented. The objective of the paper is to reduce the sidelobes of the LDB without impacting on main-lobe performance and increase the contrast-resolution of the imaging system. The Chebyshev weighting is applied to the LDB and an analytic description and the simulation results are obtained. Theoretical analysis and simulation results show that the LDB with Chebyshev weighting can reduce sidelobes, and improve imaging system performances. © *2000 Acoustical Society of America.* [S0001-4966(00)04605-1]

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INTRODUCTION

Limited diffraction beams (LDB) can propagate without changing their waveforms in both space and time provided that they are produced with an infinite aperture and energy.[1,2] Even if produced with a finite aperture, they have a large depth of field. Because of this advantage, limited diffraction beams have been used in medical imaging and many practical applications.[3,4] However, sidelobes of these beams are larger than conventional focused beams at their focuses. Sidelobes may lower the contrast in medical imaging. When the sidelobes of the LDB are reduced, the width of the main lobes are usually increased. A wide main lobe will reduce the lateral resolution of medical imaging systems.[5,6] In addition, sidelobes increase the effective sampling volume and thus average out spatially distinguished information in tissue identification. Sidelobes are also a source of multiple scattering that produces artifacts in nondestructive evaluation of materials.

In this paper, Chebyshev weighting function is used to reduce sidelobes of LDB without increasing the main-lobe width. To produce a limited diffraction beam with Chebyshev weighting in a circular two-dimensional (2D) array transducer, an aperture that produces LDB is multiplied with a Chebyshev weighting distribution array that is obtained from the Chebyshev function. A theoretical analysis for using the Chebyshev weighting method to reduce the sidelobes of LDB was developed. Simulations and analysis of the results with a finite aperture 2D annular array transducer using the Chebyshev weighting were performed. The analytical and simulation results show that LDB with Chebyshev weighting can reduce sidelobes, and thus improve the imaging system performances.

I. LIMITED DIFFRACTION BEAMS WITH CHEBYSHEV WEIGHTING DISTRIBUTION

The geometrical configuration of the transducer with Chebyshev weighting distribution is shown in Fig. 1. The spectra of limited diffraction beams (X waves and Bessel beams) with Chebyshev weighting distribution are given by

$$\Phi_{X_n f_m}\left(\mathbf{r},\frac{\omega}{c}\right)=\frac{2\pi}{c}e^{in\phi}B\left(\frac{\omega}{c}\right)J_n\left(\frac{\omega}{c}r\sin\zeta\right)H\left(\frac{\omega}{c}\right)\times e^{-(\omega/c)(a_0-iz\cos\zeta)}f(I_m)\quad (m,n=0,1,2,\ldots)\tag{1}$$

and

$$\Phi_{B_n f_m}\left(\mathbf{r},\frac{\omega}{c}\right)=J_n(\alpha r)e^{in\phi}e^{i(\beta z-\omega t)}f(I_m)\quad (m,n=0,1,2,\ldots),\tag{2}$$

respectively, where r is a radial distance, ω is angular frequency, α is a constant, m,n is an integer, $B(k)$ is the transmitting or receiving transfer function of a transducer, $J_n(x)$ is an nth-order Bessel beam function, $H(\omega/c)$ is the Heaviside step function, a_0 is the constant that determines the fall-off speed of the high frequency component of the X waves, $\beta=\sqrt{(\omega/c)^2-\alpha^2}>0$ is real, c is the phase velocity, and $f(I_m)$ is a Chebyshev weighting distribution.

Let us consider a 2D annular array of ten sources of uniform spacing d arranged as in Fig. 1. The individual sources have the amplitudes $I_0,I_1,\ldots,I_9$, etc., as indicated, the amplitude distribution being symmetrical about the center of the array. The total field Φ from the sources at a large distance in a direction θ is then the sum of the fields of the symmetrical pairs of sources, or[7]

$$\Phi_{Me}(\psi)=2\sum_{m=0}^{(M-1)}I_m\cos\left(\frac{2m+1}{2}\psi\right),\quad M\ \text{even},\tag{3}$$

$$\Phi_{Mo}(\psi)=2\sum_{m=0}^{M}I_m\cos\left[(2m)\frac{\psi}{2}\right],\quad M\ \text{odd},\tag{4}$$

where $\psi=(2\pi d/\lambda)\sin\theta$, θ is the angle shown in Fig. 1, M_e is an even number, M_o is an odd number. Each term in Eqs. (3) and (4) represents the field due to a symmetrically disposed pair of the sources.

It is well-known that $\cos m(\psi/2)$ can be expressed as a polynomial of degree m. Thus, Eqs. (3) and (4) are expressible as polynomials of degree $2k+1$ and $2k$, respectively, since each is the sum of cosine polynomials of the form $\cos m(\psi/2)$. If we now set the array polynomial as given by

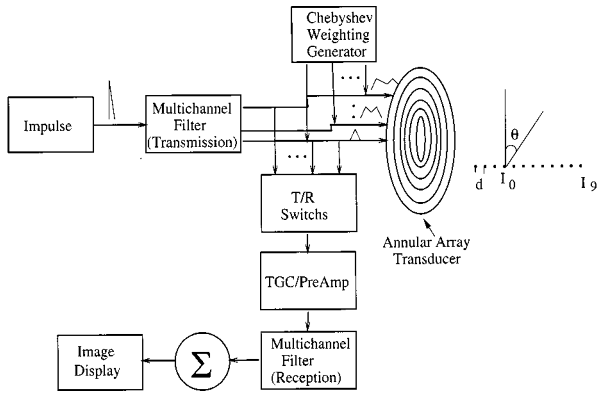

FIG. 1. 2D annular array imaging system with Chebyshev weighting distribution.

Eq. (3) or (4) equal to the Chebyshev polynomial of like degree ($m=k-1$) and equate the array coefficients to the coefficients of the Chebyshev polynomial, then the amplitude distribution given by these coefficients is a Chebyshev distribution and the field pattern of the array corresponds to the Chebyshev polynomial of degree $k-1$.

Chebyshev weighting polynomial has a binomial series form as

$$T_m(x)=\cos^m \xi-\frac{m(m-1)}{2!}\cos^{m-2} \xi \sin^2 \xi + \frac{m(m-1)(m-2)(m-3)}{4!} \times\cos^{m-4} \xi \sin^4 \xi-\ldots, \tag{5}$$

where $T_m(x)$ is Chebyshev weighting polynomials and $x=\cos \xi$. The Chebyshev function in Eq. (5) has the following properties: First, the function of all orders passes the point (1,1). Second, for values of x in the range $[-1,1]$, the polynomials lie between -1 and $+1$, and all roots of the polynomials are within $[-1,1]$.

Let the ratio of the main-lobe maximum to the sidelobe level be specified as R, i.e.,

$$R=\frac{\text{main-lobe maximum}}{\text{sidelobe level}}. \tag{6}$$

If x_0 satisfies $T_{m-1}(x_0)=R$, the point (x_0,R) on the $T_m(x)$ curve corresponds to the main-lobe maximum, while the sidelobes are confined to a maximum value of unity. The important property of the Chebyshev polynomial is that if the ratio R is specified, the beamwidth to the first null is minimized. The corollary also holds that if the beamwidth is specified, the ratio R is maximized (sidelobe level minimized).

The procedure will now be summarized. For an array of m sources, the first step is to select the Chebyshev polynomial of the same degree as the array polynomial, Eq. (3) or (4). This is given by $T_{m-1}(x)$. Next we choose R and solve $T_{m-1}(x)=R$ for x_0. The beam pattern polynomial, Eq. (3) or (4), may now be expressed as a polynomial of $\cos(\psi/2)$. The final step is to equate the Chebyshev polynomial of $T_{m-1}(x)$ and the array polynomial obtained by substituting $x=x_0 \cos(\psi/2)$ into Eq. (3) or (4), where the smallest value of ψ is 0 for $x=x_0$ (center of the main lobe). Thus,

$$T_{m-1}=\Phi_{m-1}. \tag{7}$$

The coefficients of the array polynomial are then obtained from Eq. (7), yielding the Dolph-Chebyshev amplitude distribution which is an optimum for the sidelobe level specified.

II. CHEBYSHEV WEIGHTING COEFFICIENT

An array of $m=20$ in-phase isotropic sources, spaced apart, is to have a sidelobe level 23-dB below the main-lobe maximum. Find the amplitude distribution fulfilling this requirement that produces the minimum beamwidth between first nulls, and plot the field pattern. The Chebyshev polynomial of degree $m-1$ is $T_{19}(x)$ and the value of x_0 for $T_{19}(x)=R$, where $R=15$ may be determined by trial and error from the expansion as given in Eq. (5) or may be calculated from

$$x_0=\tfrac{1}{2}[(R+\sqrt{R^2-1})^{1/(m-1)}+(R-\sqrt{R^2-1})^{1/(m-1)}]. \tag{8}$$

For $R=15$ and $m-1=19$ in Eq. (8), we have $x_0=1.016$.

Now substituting $x=x_0 \cos(\psi/2)$ in Eq. (3), we have

$$\Phi_{19}=\frac{26\,214 I_9}{x_0^{19}}x^{19}-\frac{1\,245\,184 I_9-65\,536 I_8}{x_0^{17}}x^{17} +\ldots-\frac{19 I_9-\cdots-I_0}{x_0}x. \tag{9}$$

The Chebyshev polynomial of like degree $m-1$ is given by

$$T_{19}=26\,214x^{19}-1\,245\,184x^{17}+\ldots-19x. \tag{10}$$

Now equating Eqs. (9) and (10),

$$\Phi_{19}=T_{19}. \tag{11}$$

For Eq. (11) to be true requires that the coefficients of Eq. (9) are equal to those of the terms of like degree in Eq. (10). Therefore,

$$I_9=x_0^{19}=1.352. \tag{12}$$

In a similar way we obtain

$$I_8=1.514,\ I_7=2.717,\ I_6=4.231,\ I_5=7.1,\ I_4=12.7,\ I_3=19.8,\ I_2=103,\ I_1=268,\ I_0=758. \tag{13}$$

The relative amplitudes of the ten sources are then given by

$$f(I_{(0-9)})=[1,1.12,2.01,3.13,5.24,9.42,14.65,76,198,561]. \tag{14}$$

Weighting coefficient as a function of the elements of array is given in Eq. (14). $f(I_m)$ is the Chebyshev weighting distribution that is normalized by its minimum. The values of $f(I_m)$ corresponding to a given set of values of I_m as obtained from Eq. (14) are produced with a limited diffraction beam array. If the array is a 20-element linear array, $f(I_{(0-9)})=[1,1.12,2.01,3.13,5.24,9.42,14.65,76,198,561,561,198,76,14.65,9.42,5.24,3.13,2.01,1.12,1]$ has to be produced by each element separately. If the array is a ten-element an-

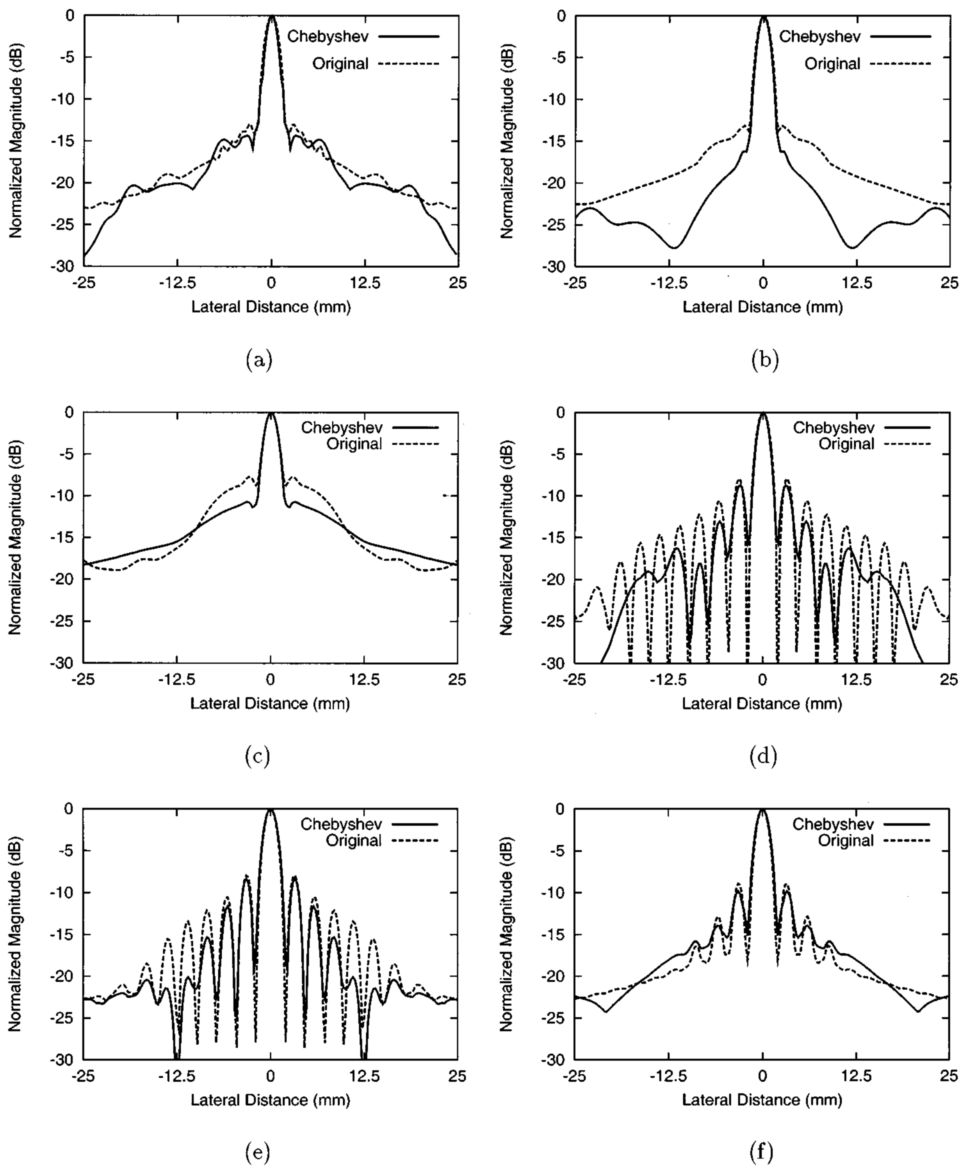

FIG. 2. Comparison between the peak lateral field responses of the original (dotted lines) and Chebyshev weighted (solid lines) limited diffraction beams. (a) X waves at $z=50$ mm. (b) X waves at $z=100$ mm. (c) X waves at $z=200$ mm. (d) Bessel beams at $z=50$ mm. (e) Bessel beams at $z=100$ mm. (f) Bessel beams at $z=200$ mm.

nular array, $f(I_{(0-9)})=[1,1.12,2.01,3.13,5.24,9.42,14.65,76,198,561]$ is produced with each element separately because an annular array is symmetric about its center point.

The operating mode of a limited diffraction beam imaging system with Chebyshev weighting is illustrated in Fig. 1. A Chebyshev weighting generator, which is controlled by a computer, is used to produce the Chebyshev weighting distribution $f(I_m)$. The excitation signals of each transducer element are multiplied with the corresponding Chebyshev weighting distribution to produce a broadband modified limited diffraction beam. Echo signals are received with the same ultrasound annular array transducer that is used in transmission.[6] The received signal from each element is connected to a T/R (transmit/receive) switch and then preamplified and compensated for attenuation with a TGC (time gain control) circuit. After multichannel filtering, the received signals are coherently summed according to the amount of charges of each ring, and images are constructed and displayed.

III. SIMULATION RESULTS

A simulation for transmission of Chebyshev weighted LDB with a ten-element ultrasonic annular array transducer was performed [use the Chebyshev weighting distribution theory given in Eqs. (1), (2), and (14), where $n=0$]. The following parameters are chosen in our simulation program: beam types are X waves and Bessel beams, orders of Bessel beams or X waves are of zero order, rotation of weightings for higher-order Bessel and X wave is 0, scaling parameter for Bessel beams is 1202.45 m^{-1}, parameter for determining high frequency decay of X waves is 0.05 mm, Axicon angle of X waves is 6.6°, selection for lateral (maximum sidelobes) beam plot, axial distances from the transducer are 50, 100, and 200 mm, respectively, use the Fresnel approximation, field radius is from 0 to 25 mm, field step size is 0.25 mm, field rotation is 0°, central frequency is 2.5 MHz, aperture stop radius is 25 mm, and weighting function is one-way Blackman window function.[2]

Figure 2(a), (b), and (c) are beam plots of the simulation results for the original limited diffraction zeroth-order X

waves and the X waves with Chebyshev weighting, with axial distances of 50, 100, and 200 mm, respectively, from the transducer. In this simulation, $B(\omega/c)$ was assumed to have the form of a Blackman window function with a central frequency of 2.5 MHz and a -6-dB one-way bandwidth about 2 MHz. From Fig. 2(a) and (b), we see that the sidelobes of the X waves with Chebyshev weighting distribution are lower than those of the original zeroth-order limited diffraction X waves. However, at $z=200$ mm [Fig. 2(c)], the sidelobes start to increase (near the boundary of the depth of field).

Figure 2(d), (e), and (f) are beam plots of the simulation results for the original zeroth-order limited diffraction Bessel beams and the Bessel beams with Chebyshev weighting produced with a finite aperture, at axial distances from the transducer of 50, 100, and 200 mm, respectively. From Fig. 2(d) and (e), we see that the sidelobes of the Bessel beams with Chebyshev weighting are lower than those of the original zeroth-order Bessel beams. Again, the sidelobes start to increase near the depth of field [200 mm, Fig. 2(f)].

In summary, from Fig. 2(a)–(f) one can see that LDB with Chebyshev weighting can reduce the sidelobes while maintaining a small beamwidth over a large depth of field.

IV. CONCLUSIONS

This paper first briefly reviewed the theory of the LDB, and pointed out the importance of reducing sidelobes of these beams. Then, a limited diffraction beam imaging system with Chebyshev weighting is proposed to reduce the sidelobes. Theoretical analysis and computer simulation were performed to study the relationship between sidelobes of LDB and Chebyshev weighting. Results show that Chebyshev weighting is effective to reduce sidelobes without increasing main-lobe width. This will increase the contrast of an imaging system while maintaining a small beamwidth (high resolution) over a large depth of field.

ACKNOWLEDGMENTS

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