A STUDY OF SIGNAL-TO-NOISE RATIO OF THE FOURIER METHOD FOR CONSTRUCTION OF HIGH FRAME RATE IMAGES

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INTRODUCTION

Limited diffraction beams were first developed in 1941 by stratton and was termed undistorted progressive waves (UPW)[1]. These beams have an infinite depth of field, i.e., they can propagate to an infinite distance without spreading, provided they are produced with an infinite large aperture. When produced with a finite aperture, they have a large depth of field[2–6]. Because of this property, limited diffraction beams have applications in medical imaging[7–10], tissue identification[11], Doppler blood flow velocity measurement[12–13], nondestructive evaluation (NDE) of materials[14], secure high-speed communications[15], and other areas such optics[6] and electromagnetics[16].

Based on the study of limited diffraction beams, recently, a new imaging method, called Fourier method, has been developed[17–20]. Unlike the conventional delay-and-sum method[21], where signals received with an array transducer are delayed and summed to focus beams on each point of an object, Fourier transform[22] is used to construct images with the new method. Because only one transmission is necessary to construct an image of an entire region of interests and the Fourier transform can be implemented with the fast Fourier transform, the new method has a potential to achieve a very high frame rate (up to 3750 m/s in biological soft tissues at a depth of 200 mm) with simplified electronics.

In this paper, we study the signal-to-noise ratio (SNR) of the new method and compare the results with the conventional delay-and-sum method. Results show that the SNR of the new method is almost identical to or a slightly better than the conventional method, provided that the same transmit beam is used.

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THEORETICAL PRELIMINARIES

Assuming a broadband transmission of a plane wave, from the X wave equation[3] one obtains receive signals from a linear array transducer weighted with a limited diffraction beam[17]

$$R_{k_x,k_y,k'_z}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{A(k)T(k)H(k)}{c} F\left(k_x, k_y, k'_z\right) e^{-i\omega t} dk, \tag{1}$$

where t is time, $R_{k_x,k_y,k'_z}(t)$ is the receive signal with the parameters of limited diffraction beams of k_x, k_y, and $k'_z = k + k_z$, where $k = \omega/c$, ω is the angular frequency, c is the speed of sound, $k_z = \sqrt{k^2 - \left(k_x^2 + k_y^2\right)}$, $F(k_x, k_y, k'_z)$ is the Fourier transform of an object function representing the ultrasound reflectivity coefficient of the object, $A(k)$ and $B(k)$ are transmitting and receiving transfer functions of the transducer, $H(k)$ is the Heaviside step function[22]

$$H\left(\frac{\omega}{c}\right) = \begin{cases} 1 & , \quad \omega \geq 0 \\ 0 & , \quad \omega < 0 \end{cases}. \tag{2}$$

By Fourier transforming the receive signal in Eq. (1), $R_{k_x,k_y,k'_z}(t)$, one obtains a relationship between the Fourier transform of receive signals and the Fourier transform of the object to be constructed. With an inverse Fourier transform, the object function can be constructed.

SIGNAL-TO-NOISE RATIO OF THE FOURIER METHOD

From Eq. (1), one can construct images from receive signals. However, in any practical systems, there is always noise. Therefore, it is important to study the influence of noise on image construction methods. In the following, the influence of noise on both the Fourier and the conventional delay-and-sum methods is studied and compared with a computer simulation.

In the simulation, a linear array transducer of 48 elements, a dimension of 18.288 mm × 12.192 mm, 2.25 MHz central frequency, and a pulse-echo bandwidth of about 81% of the central frequency was assumed[17]. A broadband ultrasound plane wave was used to illuminate a wire target whose cross section was a point. Signals received with the array transducer were superposed with a band-pass random noise whose amplitude was about 50% of the peak of the receive signals. Images were constructed from these signals with both the Fourier and the conventional delay-and-sum methods and results are shown in Figure 1.

In Figure 1, the radio-frequency (RF) signals received with the linear array with and without noise are shown on the upper right and left hand sides, respectively. The horizontal dimension is time and the vertical direction represents the number of elements. Images constructed with and without noise are shown on the lower right and left hand sides, respectively. It is seen that the SNR of the images constructed with the Fourier and the conventional methods are very similar (Figure 1).

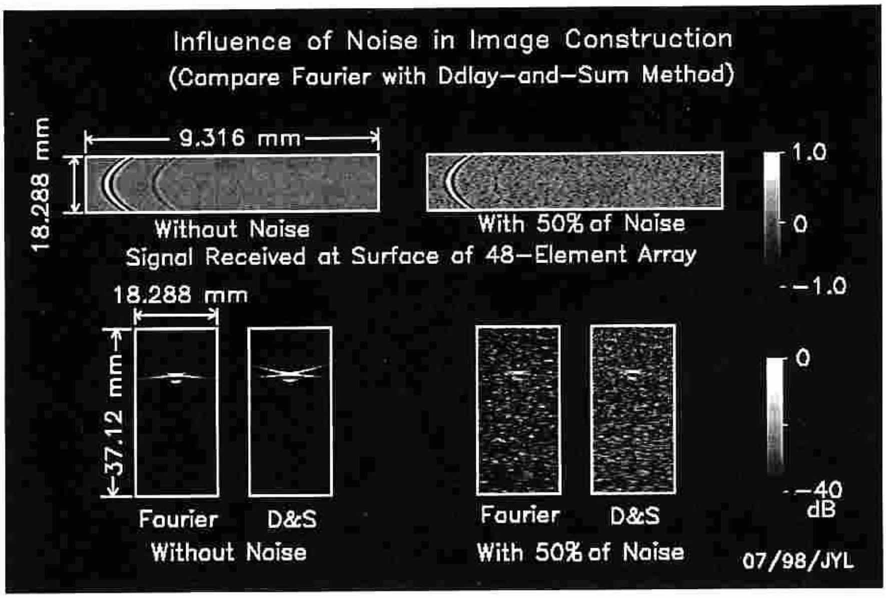

Figure 1. Influence of noise on the construction of images of a line target with the Fourier and the conventional (delay-and-sum) methods using a linear array transducer. Notice that the upper and lower grayscale bars are in linear and log scales, respectively. The speed of sound is assumed to be 1450 m/s.

DISCUSSION

The SNR is an important parameter of image quality. A higher SNR means a deeper penetration of ultrasound beams in biological soft tissues because signals from a deeper depth are weak due to tissue attenuation. From the study above, it is clear that both the Fourier and the conventional methods have a similar SNR on image constructions. Although the SNR is about the same for both the Fourier and the conventional methods, the Fourier method has the advantage that imaging systems can be greatly simplified, especially, for three-dimensional (3D) imaging where a two-dimensional (2D) array transducer is used. This is because the Fourier method can be implemented with the fast Fourier transform. As a comparison, the conventional delay-and-sum method requires to delay signals of each element and then sum the delayed signals to focus a beam at each point of an object. This process is very complex and is usually done by the so called digital beamformer.

In discussion above, a broad band plane wave is used in transmission to illuminate a large area of an object to construct high frame rate images. Because the plane wave does not diverge over the depth of interest, energy density is high as compared to a diverged beam[17]. Although a focused transmission beam may produce a higher energy density in its focal region when *f-number* is small, image frame rate is low and thus images of fast moving objects such as the heart and the blood flow within the heart will be distorted. Therefore, the Fourier method will be useful for high frame rate imaging at a high SNR.

CONCLUSION

A new image construction method, called the Fourier method, has been developed recently[17]. This method can be used to construct images at a high frame rate (up to 3750 frames/s for a depth of 200 mm in biological soft tissues) with simplified electronics because the fast Fourier transform can be used in image constructions. A computer simulation shows that the signal-to-noise ratio (SNR) of the new method is comparable to that of the conventional delay-and-sum (D&S) method when the same transmission beam is used for both methods.

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