Sidelobe Reduction of Images with Coded Limited Diffraction Beams

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Abstract – Coded excitations are studied to reduce sidelobes and increase the signal-to-noise ratio (SNR) of limited diffraction imaging systems. Images of objects consisting of randomly distributed scatterers are constructed and image contrasts are studied quantitatively. Results show that with coded excitations, high contrast images can be constructed at a high SNR.

I. INTRODUCTION

Limited diffraction beams were first described by Stratton [1] and then produced experimentally by Durnin in optics [2]. Bessel beams [1,2], X waves [3,4], and bowtie beams [5] are just a few examples of the limited diffraction beams. Theoretically, these beams have an infinite depth of field, i.e., they can propagate to infinite distance without spreading. In practice, when produced with a finite aperture and energy, these beams have a large depth of field. Because of this property, limited diffraction beams have been studied by many investigators and groups in the past decade and have been applied to both medical ultrasonic imaging [6] and optics [7].

Although limited diffraction beams have a pencil-like, localized high-amplitude center and a large depth of field, the sidelobe of these beams are higher than conventional focused beams at their focuses. High sidelobes reduce the contrast of constructed images and spread energy over a large area, reducing signal-to noise ratio (SNR) of constructed images. To reduce sidelobes of limited diffraction beams, several methods have been proposed, for example, spatial deconvolution, Wiener filtering, bowtie beam [5], and summation-subtraction [8]. Although the summation-subtraction method is very effective for sidelobe reduction, it requires three transmissions to obtain a single A-line. Therefore, it is sensitive to object motion.

Coded excitations have been studied extensively in various areas such as radar, sonar, and underwater acoustics. They were introduced to biomedical ultrasound about 10 years ago [9]. The most frequently used codes are frequency modulated (FM) chirp, pseudo-noise (PN) code, Golay code, and others. Because the codes are usually much longer than a short pulse, their total energy is large (assuming the peak amplitudes of the codes and the pulse are the same). After the codes are processed with a match filter or adaptive filter during signal processing, they can be compressed into a short pulse of a very high peak and a low range sidelobe. Because of this property, code excitations can greatly enhance the SNR of an imaging system.

In addition, some codes may have low cross correlation as well as low range sidelobes of autocorrelation. In this case, these codes can be transmitted simultaneously and then separated with autocorrelation and applied to the summation-subtraction method for reducing sidelobes of limited diffraction beams. Because each A-line (echo signal) can be obtained from a single transmission, the method will not be sensitive to object motion.

In this paper, code excitations will be combined with the summation-subtraction method to increase the SNR and reduce sidelobes of limited diffraction imaging systems (Bessel beams [1,2] are used as an example and the results will be the same for X waves [3,4]). Transmission of codes simultaneously is also studied to remove the influence of object motion.

II. PRINCIPLE OF SIDELOBE REDUCTION

The detailed theory of the summation-subtraction method for reducing sidelobes of limited diffraction pulse-echo imaging systems was given in Reference [8]. In the following, the theory is briefly reviewed. For simplicity, Bessel beams are used as an example. The results are also applicable to X waves.

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The field produced by a broadband non-rotating Bessel beams can be expressed as:

$$\Phi_{J_m} = 2\pi J_m(\alpha r)\cos m(\phi - \phi_0) F^{-1}\left\{T(\omega)e^{j\beta z}\right\}, \quad (1)$$

where $m = 0,1,2,\cdots$, is the order of the Bessel beams, $J_m(\bullet)$. $T(\omega)$ is the transmit transfer function of the transducer. $F^{-1}(\bullet)$ represents the inverse Fourier transform. From (1), one obtains the echo signal of a pulse-echo system:

$$\begin{aligned} e_{J_m}(\vec{r},t) &= 4\pi^2 A(\vec{r}) J_m^2(\alpha r)\cos^2 m(\phi - \phi_0) \\ &\bullet F^{-1}\left\{T^2(\omega)e^{j2\beta z}\right\}, \end{aligned} \quad (2)$$

where the reception transfer function is assumed to be the same as that of transmission, and $A(r,\phi,z)$ is the reflection coefficient of a point scatterer located at $\vec{r} = (r,\phi,z)$.

The combined signal from all scatterers is thus obtained:

$$\begin{aligned} e_{J_m} &= 4\pi^2 \int_{-\infty}^{\infty} r dr \int_{-\pi}^{\pi} d\phi \int_0^{\infty} dz \\ &\bullet\left[A(\vec{r}) J_m^2(\alpha r)\cos^2 m(\phi - \phi_0)\right. \\ &\left.\bullet F^{-1}\left\{T^2(\omega)e^{j2\beta z}\right\}\right]. \end{aligned} \quad (3)$$

Apparently, if $\alpha r = 0$, $J_0^2(0) = 1$ and $J_2^2(0) = 0$. This means that at the beam center, the subtraction between different orders of Bessel beams will preserve the mainlobe. On the other hand, $J_0^2(\alpha r) \approx J_2^2(\alpha r) \approx (2/\pi\alpha r)\cos^2(\alpha r - \pi/4)$ for $\alpha r \gg 1$, i.e., sidelobes of the beams are mostly cancelled after subtraction. To obtain the term, $J_2^2(\alpha r)$, two second order Bessel beams with a $\pi/4$ rotation in ϕ_0 should be summed. The sidelobes of Bessel beams can be reduced via the following summation-subtraction process:

$$\begin{aligned} & e_{J_0}(t) - [e_{J_2}(t)\,|_{\phi_0=0} + e_{J_2}(t)\,|_{\phi_0=\pi/4}] \\ &= 4\pi^2 \int_{-\infty}^{\infty} r dr \int_{-\pi}^{\pi} d\phi \int_0^{\infty} dz \\ &\bullet\left[A(\vec{r}) J_0^2(\alpha r) - J_2^2(\alpha r) F^{-1}\left\{T^2(\omega)e^{j2\beta z}\right\}\right]. \end{aligned} \quad (4)$$

For X waves, the results are similar [8]. In the following, $\alpha = 1202.45 m^{-1}$ and the diameter of transducer, $D = 50mm$, are assumed. With these parameters, the sidelobes can be reduced by about 20 dB near the edges of the transducer.

III. CODED EXCITATIONS

Coding is one of the popular techniques in radar, sonar, and telecommunication. It has also applied to medical ultrasonic imaging [9], where a long coded signal instead of a single short pulse is used to excite the transducer. Because the damage threshold of human tissues limits the peak intensity of the transmitted ultrasound beams, the total energy of a short pulse is small, which in term, limits the penetration depth and the SNR of the imaging system. With coded excitations, the energy of sound beams is spread over a longer period of time, and thus the total energy is usually much higher than that of a short pulse. In reception, the long code can be compressed into a short pulse of a high peak intensity to increase the penetration depth and the SNR of the ultrasound signals.

We have studied several types of codes, such as, *m*-sequence, FM chirp, and Golay code pairs [9]. *m*-sequence is one type of pseudo-noise code possessing a noise-like spectrum. The autocorrelation of an *m*-sequence is determined by the length of the code. With a 127-chirp *m*-sequence, the range sidelobe of the autocorrelation is about -40dB relative to the peak. However, this is correct in the sense of circular correlation, which is difficult to realize in practice. When implemented with a linear correlation, the range sidelobes are increased to about -20 dB. The cross-correlation between two *m*-sequences depends on which sequence is selected in addition to the code length. Computer simulation was used to find three *m*-sequences of 127 bits with low mutual cross-correlation (about −18 dB) by searching all the possible combinations of the sequences. These three *m*-sequences are used in the summation-subtraction method and transmitted simultaneously. To make a full use of the bandwidth of the transducer, two neighboring bits last for one cycle at the central frequency. Unfortunately, the range sidelobes of these *m*-sequences are too high for biomedical imaging.

One alterative to *m*-sequence is FM chirp, which can be easily designed to make use of the full bandwidth of the transducer. With matched filtering, the range sidelobe of a windowed FM chirp can be lower than –50 dB. If the bandwidth of a transducer is wide enough, three orthogonal FM chirps can be produced by dividing the bandwidth into three equal parts. The mutual cross-correlation of these chirps will be very low, provided that there is no overlap between the spectra of the chirps. Unfortunately, this approach reduces the axial resolution because of decreased bandwidth of each chirp and the frequency dependent attenuation of tissues may be a problem for the summation-subtraction method. Therefore, in this paper, only single FM chirp occupying the entire bandwidth is studied. Unlike the conventional imaging systems where dynamic focusing is used, the length of the FM chirp for a limited diffraction imaging system is limited only by the acceptable range of the dead zone.

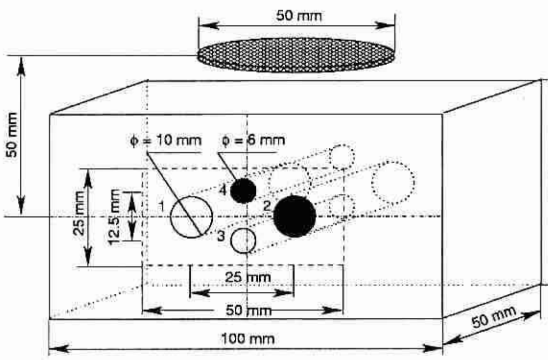

Fig.1. Schematic of scattering phantom. The diameters of cylindrical objects 1, 2, 3, 4 are of 10, 10, 6, and 6 mm, respectively, and their contrasts relative to the background are 15, -15, 15, and –15 dB.

Golay code pairs have very low range sidelobes of autocorrelation. When two autocorrelations of a Golay pair are added together, they produce a δ -function. The main disadvantage of Golay pairs is that it needs multiple transmissions to achieve pulse compression, thus it is sensitive to motion. Because of this, Golay pairs are not used in the summation-subtraction method to reduce sidelobes.

IV. RESULTS

In computer simulation, a 2.5 MHz center frequency, 10-ring annular transducer [10] of a diameter of 50 mm and a –6 dB bandwidth of about 81% of the central frequency is assumed. The 10 rings are arranged according to the first 10 zeros of $J_0(\alpha r)$, where $\alpha = 1202.45 m^{-1}$. To transmit second order Bessel beams, each ring is subdivided into 40 equal segments.

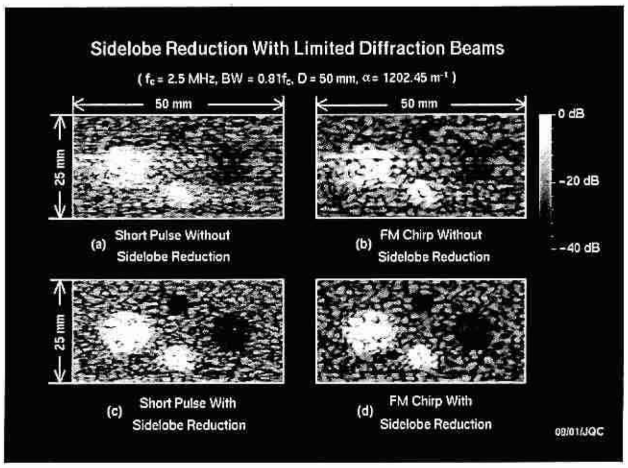

Fig.2. Constructed images of the phantom in Fig.1.

The schematic diagram of the 3 D scattering phantom is shown in Fig.1. The scatterers are randomly distributed and, in average, there is one scatterer in each cubic wavelength, i.e., there are about 1.2 million scatterers in total.

Phantom images constructed with a short pulse and FM chirp are shown in Fig. 2. Panels (a) and (c) are images obtained with a short pulse and Panels (b) and (d) are those with an FM chirp. The top two panels in Fig. 2 are images obtained without using the summation-subtraction sidelobe reduction (use a zeroth-order Bessel beam only) and images in the lower two panels are with the sidelobe reduction.

Table 1 Contrast of Cylindrical Objects in Fig. 2

Contrast (dB)	Obj. #1	Obj. #2	Obj. #3	Obj. #4	Bg.
Ideal	15	-15	15	-15	0
(a)	12.5	-6.9	12.7	-5.2	0
(b)	12.6	-6.3	12.7	-5.5	0
(c)	14.7	-13.1	16.1	-11.4	0
(d)	15.0	-14.0	17.0	-10.5	0

Contrasts of the cylindrical objects are shown in Table 1. The labels (a), (b), (c), and (d) correspond to those in Fig. 2. The contrasts are calculated with the formula:

$$contr = 20\log\frac{m_i}{m_o}, \qquad (5)$$

where, m_i is the mean of the constructed image of the cylinders and m_o is the mean of the background. +/-1 mm areas around the edges of the cylinders are not included in the contrast calculation to avoid the influence of the edges (+/-1 mm is chosen because the beam width is close to 2 mm).

V. DISCUSSION

The simulation results in Fig. 2 and Table I demonstrated that the summation-subtraction method can reduce sidelobes dramatically, as predicted by the theory in Section II.

Comparing the results of a short pulse ((a) and (c) in Fig.2) and an FM chirp ((b) and (d) in Fig.2), it is seen that sidelobe reduction is effective in both cases. This means that sidelobes can be reduced while the code excitation increases the SNR. The slightly bigger speckle sizes in the code excitation are due to the window applied to the code.

To reduce effects of motion in the summation-subtraction method, we have used three 127-chip *m*-sequences to encode the three beams and transmit them simultaneously. Unfortunately, because of the high range sidelobes of the autocorrelations and poor cross-correlation, the contrasts of constructed images are low. Therefore, it is important to find codes with low cross correlation and high autocorrelation peak if multiple beams are to be transmitted simultaneously.

VI. CONCLUSION

Coded excitations are studied for sidelobe reduction of limited diffraction imaging systems. If codes of low cross-correlation and high autocorrelation peak can be found, high contrast and high SNR images can be constructed even for fast moving objects.

VII. ACKNOWLEDGEMENTS

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