# Recursive Fourier-Based High-Frame Rate Imaging

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Abstract - The high-frame-rate (HFR) imaging method developed in 1997 used broad beams such as steered plane waves or limiteddiffraction beams to illuminate an object, and then received echo signals are used to reconstruct images with Fourier transform. This method can be used in fast ultrasound cardiac imaging, flow velocity vector imaging, elasticity imaging, strain and strain rate imaging, and functional imaging. However, when multiple transmissions are used to reconstruct a frame of image to increase image field of view and improve image quality, image frame rate is reduced. To achieve the highest image frame rate limited only by the ultrasound round-trip time while multiple transmissions are used to reconstruct each frame of image, a recursive method is used. In this method, images reconstructed from each transmission are weighted with a constant that is between 0 and 1, and then added to the most recently reconstructed image. Experiments were performed with a tissuemimicking phantom to demonstrate the efficacy of the method. Results show that high-quality images can be reconstructed at the highest image frame rate that is usually achievable only when one transmission is used to reconstruct a frame of image.

Keywords - Recursive imaging, high-frame-rate imaging, fast Fourier transform, plane wave and diverging wave, limiteddiffraction beams

#### I. INTRODUCTION

High-frame-rate (HFR) ultrasound imaging [1]-[6] is important for elasticity imaging of moving objects [7]-[9], blood flow velocity vector imaging [10]-[13], fast cardiac imaging [5][9][14], strain and strain rate imaging [14][15], and functional imaging [16]. Previously, a Fourier-based HFR imaging method that uses a single plane wave [1][2], steered plane waves (SPW) [1]-[4], steered diverging waves (SDW) [17]-[18], or limited diffraction beams (LDB) [5][6][11][12][19]-[21] in transmissions (TXs) has been developed. This method has advantages that Fast Fourier transform can be used in image reconstructions to reduce computations [1][2][5][6], and when limited-diffraction beams are used in TXs, only one or two high-voltage transmitters are needed to drive all elements of an either 1D or 2D array transducer [5]. Despite the advantages, image frame rate is reduced when multiple TXs are used to reconstruct a frame of image to increase image field of view and to improve image quality [3]-[6].

In this paper, a recursive method used in the synthetic aperture imaging [22] is applied to the HFR imaging method [3]-[6] to achieve a high image frame rate when multiple

transmissions are used to increase image filed of view and improve image quality.

#### II. METHOD

To achieve the highest image frame rate (reciprocal of one ultrasound round-trip time) for the Fourier-based HFR imaging method [1]-[6] when using multiple transmissions per frame, a recursive method [22] is used, where the sub-image obtained from each transmission is recursively added to the most recently reconstructed image.

The recursive formula used is as follows [22]:

$$f_i(\vec{r}) = c_0^{i+1} f_{i+1}(\vec{r}) , \qquad (1)$$

where  $\vec{r} = (x, y, z)$  is a point in space in the Cartesian coordinates,  $0 < c_0 < 1$  is a constant,  $i = 0, 1, 2, \cdots$  is the frame number, and  $f_i(\vec{r})$  and  $f_{i+1}(\vec{r})$  are the *i* th and (i+1) th frames of images, respectively. The image that includes the contributions of the first  $n \ge 0$  frames of images is given by the following superposition of weighted frames of images:

$$F_n(\vec{r}) = \sum_{i=0}^n c_0^{n-i} f_i(\vec{r}).$$
 (2)

Since  $0 < c_0 < 1$ , if each frame of image is identical (no object motion between frames), i.e.,  $f(\vec{r}) = f_0(\vec{r}) = f_1(\vec{r}) = f_2(\vec{r}) = \cdots$ , the summation in Eq. (2) will converge as  $n \to \infty$  since it is a geometrical series.

If each frame of image is reconstructed with  $m \ge 1$  transmissions, then, each frame of image is formed by the following weighted superposition formula:

$$f_i(\vec{r}) = \sum_{j=0}^{m-1} k_0^{(m-1)-j} f_{ij}(\vec{r}), \qquad (3)$$

where  $f_{ij}(\vec{r})$  is the image obtained from the *j* th transmission for the *i* th frame of image,  $j = 0, 1, 2, \dots, m-1$ , and  $k_0 = e^{(\ln c_0)/m}$  is also a constant. Since  $0 < c_0 < 1$ , we have  $0 < c_0 \le k_0 < 1$ . When m = 1, we have  $k_0 = c_0$ . Inserting Eq. (3) into Eq. (2), we have the following superposition of weighted images obtained from individual transmissions:

$$F_n(\vec{r}) = \sum_{i=0}^n c_0^{n-i} \left[ \sum_{j=0}^{m-1} k_0^{(m-1)-j} f_{ij}(\vec{r}) \right].$$
(4)

# **ATS539 Tissue Mimicking Phantom**



Figure 1. A cross-section of an ATS539 tissue-mimicking phantom showing the imaging areas of 120 mm depth with +/-45 degree field of view for the experiments. (Modified from Fig. 4 of [24].).

## III. EXPERIMENT

To test the method, a home-made HFR imaging system [5][6][23] with a phased array transducer of 128-elements, 2.5-MHz center frequency, 19.2-mm aperture, and 58% -6dB pulse-echo bandwidth was used to acquire data from an ATS539 tissue-mimicking phantom (Fig. 1). Images of 90-degree field of view and 120 mm depth were reconstructed with the Fourier-based method [1][2].



# Image Reconstruction with and without Recursion

Figure 2. Images reconstructed with (first row) and without (second row) the recursive method from echo signals acquired experimentally. The frame recursive constant  $C_0 = 0.5$ . All images are reconstructed with the Fourier-based high-frame-rate (HFR) imaging method [1]-[6]. Images reconstructed with the steered plane wave (SPW) and the limited-diffraction beams (LDB) are in Panels (a)-(d) and (e)-(f), respectively. Results of 11 transmissions are shown in Panels (a), (b), (e), and (f), while those of 91 transmissions are in Panels (c), (d), (g), and (h). 50 dB log compression has been applied.

### IV. RESULTS

Images reconstructed with and without the recursive method when the frame recursive constant  $c_0 = 0.5$  (the weight between successive frames of images) are shown in the first and second rows of Fig. 2 respectively. Images in the 1st and 3rd columns are obtained with 11 TXs per frame and the rest are obtained with 91 TXs per frame. The left and right four images are obtained with the SPW and LDB transmissions respectively. The results show that the quality of images obtained recursively is maintained while achieving a frame rate of 5347 (187 us between TXs). The same result can also be seen from Table 1 where the contrasts of the 4<sup>th</sup> cystic object (see the arrow in Fig. 1) with and without using the recursive method are compared.

The relatively small  $c_0$  makes the contributions of prior frames of images small (decays as a power of  $c_0$ ), reducing motion artifacts. For example, given  $c_0 = 0.5$  and i = 6, the influence of a frame of image  $f_0(\vec{r})$  is only  $c_0^6 = 0.015625 \approx 1.6\%$  of that of the frame of image  $f_6(\vec{r})$ .

**Table 1.** Contrasts (20 times the logarithm of the average pixel values inside the center circle in the cyst over that between the two outer rings in the background) of the images of the 4<sup>th</sup> large cyst (8 mm in diameter) from the top of the ATS539 tissue-mimicking phantom with and without the recursive method.

Cyst Contrast (dB)	11 Transmissions		91 Transmissions	
	SPW	LDB	SPW	LDB
Recursive	-14.01	-14.62	-24.00	-24.85
Normal	-13.99	-14.88	-24.11	-24.91

#### V. DISCUSSION AND CONCLUSION

Application of the recursive method [22] to the Fourierbased high-frame-rate (HFR) imaging method [1]-[6] can achieve the highest image frame rate (limited only by the round-trip time of ultrasound) when multiple transmissions are used to reconstruct each frame of image. This allows the HFR images to be zoomed in time at a very high time resolution to track motion of fast moving objects, which will be useful for cardiac imaging and its image analyses. With the recursive method, parameter images such as velocity vector or elasticity obtained from the HFR imaging can also be produced at the highest image frame rate.

In addition to the SPW and LDB imaging results in this paper, the recursive method can also be applied to the steered diverging wave (SDW) transmissions with small diverging angles [17].

When the frame recursive constant  $c_0$  is close to 1, each frame of image will be more uniform from side to side. In contrast, as  $c_0$  is approaching 0, each frame of image will be uneven from side to side. As can be seen from the results in this paper, with  $c_0 = 0.5$ , the images are reasonably uniform, although the images are a little darker in the edges as compared to those obtained without using the recursive method.

Despite the uniformity of each frame of image when  $c_0$  is close to 1, motion artifacts [25][26] will occur. This is because the earlier frames of images will have longer lingering effects to the current image. If objects move, the traces of motion will be shown, which will not only create image artifacts, but also lower image resolution of moving objects.

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